

## Anisotropy of the hole drift mobility in barium titanate

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The static dielectric tensor is highly anisotropic in the tetragonal phase of barium titanate: the static dielectric constant perpendicular to the  $c$  axis,  $\epsilon_{\perp}$ , is 40 times that parallel to the  $c$  axis,  $\epsilon_{\parallel}$ , at  $T=20$  °C, with this ratio decreasing to  $\sim 20$  times at  $T=90$  °C. Using both holographic and bulk measurements we show that the hole drift mobility has comparable anisotropy: the hole mobility perpendicular to the  $c$  axis,  $\mu_{\perp}$ , is 20 times larger than that parallel to the  $c$  axis,  $\mu_{\parallel}$ , at  $T=20$  °C, with this ratio decreasing to 8 times at  $T=90$  °C. We also show that  $\mu_{\parallel}/\mu_{\perp} \sim 2\epsilon_{\parallel}/\epsilon_{\perp}$  independent of temperature between  $T=20$  °C and  $T=90$  °C, suggesting that the hole drift mobility in barium titanate crystals scales with the static dielectric constant in this temperature range.

Between  $\sim 10$  and  $\sim 125$  °C barium titanate is a tetragonal, ferroelectric insulator, with highly anisotropic dielectric and electro-optical properties. In 1986, Tzou, Chang, and Hellwarth found that the mobility of holes in BaTiO<sub>3</sub> crystals was also highly anisotropic at room temperature: the mobility perpendicular to the  $c$  axis,  $\mu_{\perp}$ , was 20 times larger than the mobility parallel to the  $c$  axis,  $\mu_{\parallel}$ .<sup>1</sup> Curiously, the static dielectric tensor has the same slant; the dielectric constant perpendicular to the  $c$  axis,  $\epsilon_{\perp}$ , is larger than the dielectric constant parallel to the  $c$  axis,  $\epsilon_{\parallel}$ , by a factor of 40. Although the anisotropy of the crystal properties is not unexpected, it is surprising that the hole drift mobility is higher in the direction with the higher dielectric constant and by nearly the same factor. Is this a coincidence, or is the mobility proportional to the dielectric constant in barium titanate?

One way to investigate a possible correlation between the mobility and the dc dielectric constant in barium titanate is to see how these quantities vary with temperature. Barium titanate has phase transitions at 125 and 10 °C that are associated with soft phonon modes polarized parallel or perpendicular to the  $c$  axis, so the dielectric constants show a large variation over this temperature range. We therefore extended Tzou, Chang, and Hellwarth's measurements to the temperature range  $T=20$ – $90$  °C, over which the ratio of the dielectric constants varies by a factor of 4;  $\epsilon_{\perp}$  decreases by a factor of 2, while  $\epsilon_{\parallel}$  increases by a factor of 2. We found that the ratio of mobilities parallel and perpendicular to the  $c$  axis indeed follows the ratio of dielectric constants in the two directions;  $\mu_{\parallel}/\mu_{\perp} \sim 2 \times \epsilon_{\parallel}/\epsilon_{\perp}$  over the measured temperature range. To ensure that the measured variations are not an impurity-dependent effect, we repeated these experiments in three barium titanate samples with widely different photoconductive properties. The same relation held in all three. Our results suggest that there is some fundamental connection between the static dielectric constant and drift mobility in barium titanate. To the best of our knowledge, no theory predicts this.

Drift mobility values derived from a measurement of photoconductivity are suspect because they depend on a precise knowledge of the density of carriers. Here we remove this ambiguity by measuring the ratio of mobilities, which is independent of the carrier density. We measured the hole mo-

bility ratio  $\mu_{\perp}/\mu_{\parallel}$  in barium titanate crystals using two methods: a straightforward measurement of the crystal's photoconductivity using electrodes placed on the crystal, and a holographic measurement that required no electrodes and instead used the photorefractive effect.

In our holographic experiments we interfere two light beams inside the crystal to create a spatially periodic intensity pattern. The light rearranges charges between deep trapping sites in a crystal. Impurities, vacancies, or defects in the crystal act as charge donors and acceptors. In nominally undoped barium titanate crystals, holes are the dominant charge carriers.<sup>2</sup> Once excited by light, the holes drift and diffuse through the crystal until they are retrapped. The holes are excited out of the bright regions and collect in the darker regions. The dc electric field of the resulting periodic charge pattern distorts the crystal lattice, and so creates a spatially periodic change in the crystal's index of refraction by the Pockels effect. We monitor the strength of this refractive-index grating by its diffraction of a separate light beam incident at the Bragg angle. We then make this grating decay by exposing it to an intense, uniform beam of light while recording its decay with time.

According to the single-trap-level theory of the photorefractive effect, at a temperature  $T$  the erasure rate of a photorefractive grating illuminated by a uniform beam of intensity  $I$  is given by<sup>3</sup>

$$\Gamma = \frac{e\mu n_0}{\epsilon} (1 + k_g^2/k_0^2), \quad (1)$$

where  $\mu(T)$  is the microscopic mobility,  $\epsilon(T)$  is the dielectric constant,  $k_0(T)$  is the Debye screening wave vector along the direction of charge migration,  $k_g$  is the magnitude of the grating wave vector, and  $e$  is the electric charge. The intensity dependence of Eq. (1) is hidden in the factor  $n_0(I, T)$ , the density of photocarriers. Equation (1) is approximately valid even if more than one trap level contributes to the crystal's photoconductivity, as long as  $n_0$  and  $k_0$  are calculated using a multiple-trap model.<sup>4</sup>

We measured the light-induced erasure rates  $\Gamma$  of photorefractive gratings oriented parallel or perpendicular to the  $c$  axis at a fixed light intensity and as a function of tempera-

ture, in three well-characterized BaTiO<sub>3</sub> samples. We previously measured values of the Debye screening wave vector  $k_0$  for both orientations of these three crystals at room temperature. In the present experiments we chose the grating wave vector  $k_g = 1.49 \mu\text{m}^{-1}$  to be small, so that the quantity in parentheses in Eq. (1) would be insensitive to variations in  $k_0$  with temperature. From our measurements of  $k_0$  and the grating erasure rates  $\Gamma$ , we determined the dielectric relaxation rates  $e\mu n_0/\varepsilon$  parallel and perpendicular to the crystal's  $c$  axis. For both the  $\mathbf{k}_g \parallel \hat{\mathbf{c}}$  and  $\mathbf{k}_g \perp \hat{\mathbf{c}}$  experiments, the erasing light had the same intensity and was polarized to be perpendicular to the  $c$  axis, so that the density of free carriers  $n_0(I, T)$  was the same for both crystal orientations. Consequently,  $n_0(I, T)$  conveniently cancels out when we take the ratio  $R$  of the measured dielectric relaxation rates parallel and perpendicular to the  $c$  axis:

$$R = \frac{\mu_{\parallel}/\varepsilon_{\parallel}}{\mu_{\perp}/\varepsilon_{\perp}} = \frac{\Gamma_{\parallel}(1 + k_g^2/k_{0\perp}^2)}{\Gamma_{\perp}(1 + k_g^2/k_{0\parallel}^2)}. \quad (2)$$

We used four-wave mixing to measure the photorefractive grating decay rates  $\Gamma_{\parallel}$  and  $\Gamma_{\perp}$  in three well-characterized BaTiO<sub>3</sub> crystals named Swiss, Free, and Chip, chosen for their different photoconductive properties. The Swiss crystal has a small dark conductivity (dark-storage time  $> 8$  h), and a photoconductivity  $\sigma_{\text{photo}} \sim I^{0.9}$  that scales nearly linearly with intensity at room temperature.<sup>5</sup> In contrast, the Free crystal has a relatively large dark conductivity (dark-storage time  $\sim 3$  s), and a photoconductivity that scales as  $\sigma_{\text{photo}} \sim I^{0.68}$  at room temperature.<sup>5</sup> The Chip crystal has a long dark-storage time and a photoconductivity that scales as  $\sigma_{\text{photo}} \sim I^{0.9}$ ; however, unlike the other two crystals, it shows hologram fixing.<sup>6</sup> The distinct photoconductive properties of these samples are determined by the densities and types of deep and shallow traps in each crystal.

The erasure rates were measured as follows. Two coherent writing beams intersected in the crystal at an angle  $2\theta = 7^\circ$  in air. This crossing angle is large enough so that effects of photogalvanic currents are negligible,<sup>7</sup> and small enough so that the grating wave vector is much smaller than the Debye screening wave vector. At room temperature in these crystals, we obtained  $(1 + k_g^2/k_{0\parallel}^2) = 1.0$  and  $(1 + k_g^2/k_{0\perp}^2) = 1.4$  for the Swiss crystal, and  $(1 + k_g^2/k_{0\parallel}^2) = 1.0$  and  $(1 + k_g^2/k_{0\perp}^2) = 1.2$  for the Free crystal.<sup>8</sup> For the Chip crystal  $(1 + k_g^2/k_{0\parallel}^2) \sim 1.0$ , and  $(1 + k_g^2/k_{0\perp}^2) = 1.2$ . An intense erasing beam, incoherent with the writing beams, flooded the crystal at all times. After a photorefractive grating was written to steady state, we caused the photorefractive grating to decay by rapidly vibrating one of the writing beam's mirrors, thereby ruining the interference pattern inside the crystal. This trick of keeping the erasing beam on continuously eliminated large transients in the photoconductivity of the crystal, although it decreased the peak strength of the photorefractive grating. The crystal was oriented so as to make the grating wave vector either parallel or perpendicular to the  $c$  axis. The intensities of the writing beams were  $I_1 = 0.2 \text{ W/cm}^2$  and  $I_2 = 0.049 \text{ W/cm}^2$ , and the intensity of the erasing beam was  $I = 4.0 \text{ W/cm}^2$ . All of these optical beams were at 514.5 nm and polarized perpendicular to the  $c$  axis of the crystal for both orientations of the crystal. The crystal

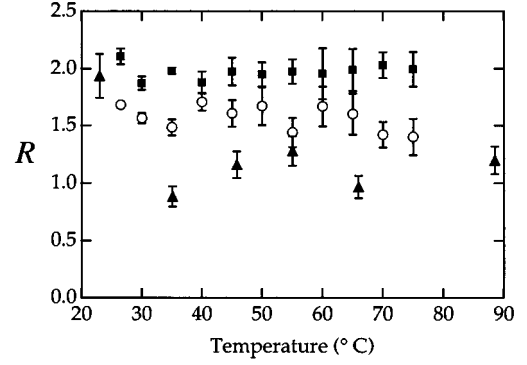


FIG. 1. Ratio  $R$  of the dielectric relaxation rates parallel and perpendicular to the  $c$  axis vs temperature in three barium titanate crystals: Swiss (full squares), Free (open circles), and Chip (full triangles).

temperature was stabilized to  $\pm 0.2^\circ\text{C}$ . The photorefractive grating was monitored by a weak, extraordinary-polarized beam from a He-Ne laser at  $\lambda = 632.8 \text{ nm}$  incident at the Bragg angle.

For the Swiss and Free crystals the decay of the diffracted signal fit well to a single exponential decay, and the erasure rate  $\Gamma$  was taken to be the inverse of the  $1/e$  decay time. For the Chip crystal the grating decay was best fit to a double exponential decay, with the two time constants differing by a factor of 100 or more. (The slower decay is thought to be caused by ionic motion.<sup>6</sup>) Here we take the erasure rate as the inverse of the  $1/e$  time constant of the faster decay.

Figure 1 shows the ratio  $R$  of the measured decay rates for gratings oriented parallel and perpendicular to the  $c$  axis in our three barium titanate crystals as a function of temperature. Note that  $R$  is approximately *independent* of temperature in this temperature range. For the Swiss crystal  $R_{\text{Swiss}} = 2.0 \pm 0.1$ , and for the Free crystal  $R_{\text{Free}} = 1.6 \pm 0.1$ . In the Chip crystal  $R_{\text{Chip}} = 1.1 \pm 0.1$  between 35 and  $90^\circ\text{C}$ , but deviates to  $R \sim 2$  for  $T = 23^\circ\text{C}$ .

It surprised us that  $R$  was relatively constant with temperature even though, according to the literature<sup>9</sup> and to our experiments, both of the dielectric constants in Eq. (2) change markedly with temperature. Consequently, we performed an independent measurement of  $R$  by separately measuring the dielectric constant and the photoconductivity parallel and perpendicular to the  $c$  axis in one of the barium titanate crystals. We attached silver paste electrodes to opposite faces of the Chip crystal and applied  $\sim 5\text{--}50 \text{ V}$  while illuminating it from the side by an  $I = 0.29 \text{ W/cm}^2$ , ordinary-polarized optical beam at 514.5 nm. The crystal temperature was controlled to better than  $\pm 0.1^\circ\text{C}$ . At each temperature and before each measurement the crystal was allowed to come to equilibrium to ensure that any transient pyroelectric currents had thoroughly decayed. The photocurrent through the crystal was measured by a Keithley Model 617 programmable electrometer. To avoid complications from photogalvanic currents when measuring the photoconductivity along the  $c$  axis, we measured the differential conductivity as follows. The applied voltage was slowly stepped at 5-V intervals from  $-50$  to  $+50 \text{ V}$  and back to  $-50 \text{ V}$ , and the current was measured. The conductivity of the crystal was deter-

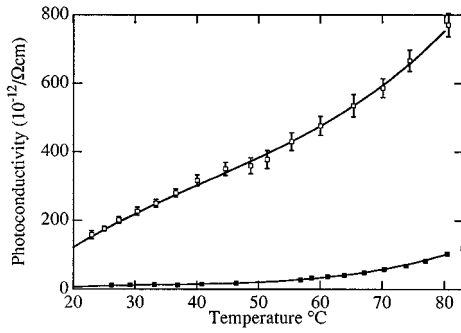


FIG. 2. Measured photoconductivity  $\sigma = e\mu n_0$  in the Chip crystal of barium titanate: parallel to the  $c$  axis (full squares), perpendicular to the  $c$  axis (empty squares). The lines are curve fits, which we use to generate Fig. 4.

mined from the slope of the current versus voltage ( $I$ - $V$ ) curve. We used the same method to measure the conductivity perpendicular to the  $c$  axis. The  $I$ - $V$  curve for the crystal was linear for applied voltages less than 100 V. The static dielectric constant was measured by applying a voltage to the crystal in the dark, assuming that it formed a parallel plate capacitor, and measuring the accumulated charge with the electrometer.

Figure 2 shows the measured values for the photoconductivity  $\sigma = e\mu n_0$ , and Fig. 3 the dielectric constant  $\varepsilon$  both parallel and perpendicular to the  $c$  axis in the Chip crystal. Note that the photoconductivity perpendicular to the  $c$  axis is larger than the photoconductivity parallel to the  $c$  axis; the photoconductivity is larger in the direction having the larger dielectric constant. This agrees with our observation that the photorefractive erasure rate is nearly the same for gratings aligned parallel and perpendicular to the  $c$  axis. The increase in the photoconductivities along both directions with increasing temperature is caused in part by the increase in the photocarrier density with temperature.<sup>5</sup> Hence the temperature dependence of mobilities cannot be determined from these data.

Figure 4 compares the ratio  $R$  of the dielectric relaxation rates  $e\mu n_0/\varepsilon$  parallel and perpendicular to the Chip crystal's  $c$  axis computed by two different methods. We obtain the triangular data points from our grating decay measurements (Fig. 1). We obtain the shaded region from our bulk measure-

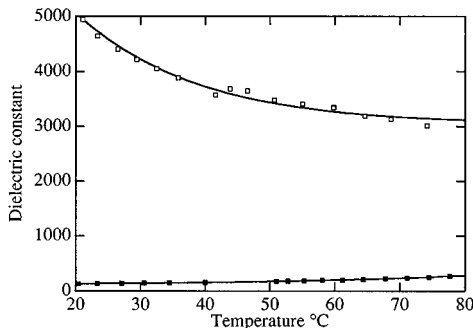


FIG. 3. Measured dc dielectric constant  $\varepsilon$  in the Chip crystal of barium titanate parallel to the  $c$  axis (full squares), perpendicular to the  $c$  axis (empty squares). The lines are curve fits, which we use to generate Fig. 4.

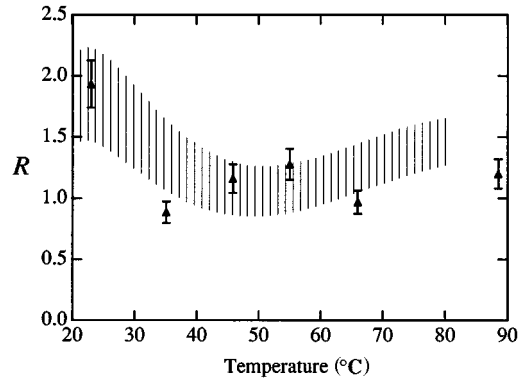


FIG. 4. The ratio  $R$  of dielectric relaxation rates  $e\mu n_0/\varepsilon$  parallel and perpendicular to the  $c$  axis of the Chip crystal: holographic measurement (full triangles); bulk measurement (shaded region).

ments of the photoconductivities and dielectric constants; we empirically fit smooth curves through the data in Figs. 2 and 3, and then formed the ratio of these four curves to obtain  $R_{\text{bulk}} = \sigma_{\parallel} \varepsilon_{\perp} / \sigma_{\perp} \varepsilon_{\parallel}$ . The vertical extent of the shaded region in Fig. 4 reflects the uncertainties in our measurements of the photoconductivities and the dielectric constants. The holographic and electrical measurements are in good agreement.<sup>10</sup>

We did not repeat the electrical measurements of  $R$  in the other two crystals because this method is more difficult and, since it involves the surface of the crystal, less reliable than the holographic technique. Also, electrical measurements require good crystal surface quality, which was lacking on one crystal.

Our data in three different samples of barium titanate give a ratio  $R$  of dielectric relaxation rates that is relatively independent of temperature over the range  $T = 20$ – $90$  °C, even though in this temperature range  $\varepsilon_{\parallel}$  doubles while  $\varepsilon_{\perp}$  decreases by a factor of 2. To keep the ratio  $R$  independent of temperature, the mobilities must behave according to

$$\frac{\mu_{\parallel}(T)}{\mu_{\perp}(T)} = R \times \frac{\varepsilon_{\parallel}(T)}{\varepsilon_{\perp}(T)}. \quad (3)$$

From these data we conclude that the hole mobility is proportional to the dielectric constant, thereby keeping the ratio  $R$  relatively independent of temperature. It is interesting to note that Wemple, Di Domenico, and Jayaraman have found the seemingly opposite trend,  $\sigma = 1/\varepsilon + \text{const}$ , in cubic  $n$ -type semiconducting barium titanate at 140 °C.<sup>11</sup> They arrived at this conclusion indirectly by showing that conductivity increased while dielectric constant decreased with increasing hydrostatic pressure. They explained their results based on a band picture with electron–TO-phonon interaction as the dominant scattering mechanism that determines the mobility. They did not carry out measurements or make predictions for the tetragonal phase of barium titanate.

Figure 5 shows a separate measure of the ratio of mobilities  $\mu$  in the Chip crystal; here we measure the photoconductivities directly in the two directions and plot their ratio  $\sigma_{\perp}/\sigma_{\parallel} = (e\mu_{\perp}n_0/e\mu_{\parallel}n_0) = \mu_{\perp}/\mu_{\parallel}$ . This ratio decreases from a value of 20 at  $T = 20$  °C to a value of 8 at  $T = 90$  °C, so the mobilities appear to approach the same value as the temperature is increased toward the tetragonal-

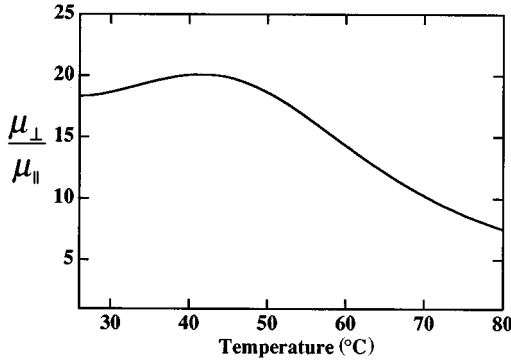


FIG. 5. The ratio  $\mu_{\perp}/\mu_{\parallel}$  of hole mobilities perpendicular and parallel to the  $c$  axis computed from the ratio  $\sigma_{\perp}/\sigma_{\parallel}$  of the measured photoconductivities in the Chip crystal of barium titanate.

to-cubic phase transition at  $T \sim 125$  °C. Previous conductivity and Hall measurements by Berglund and Baer<sup>12</sup> in  $n$ -type conducting barium titanate showed a temperature-dependent anisotropy for electron drift mobility that is similar to our results for photogenerated holes.

The anisotropy in the hole drift mobility in barium titanate and its apparent connection with the dielectric constant is not satisfactorily explained by any theory. All of the previous work on mobilities in barium titanate is on electron transport in semiconducting crystals,<sup>11–14</sup> and, even then, there is no consensus on a theory. Berglund and Baer explain the electron mobility anisotropy based on a many-valley band conduction model,<sup>12</sup> while Girshberg, Bursian, and Grushevsky explain the same results based on a small-polaron hopping model.<sup>13</sup> As we explain below, neither model seems satisfactory for hole transport.

Band transport is not an appropriate description for hole conduction in barium titanate. According to band-transport models, a charge is assumed to move as a free particle of effective mass,  $m^*$ , in the crystal between collisions with phonons. However, this picture breaks down for crystals with low carrier mobility where the “scattering” length calculated from the Drude model,  $l_s \sim m^*v\mu/e$  becomes comparable to the de Broglie wavelength of the electron,  $\lambda_D \sim h/m^*v$ . Assuming the carrier is moving at the thermal velocity,  $v \sim \sqrt{3k_B T/m^*}$ , and taking its effective mass to be the free-electron mass, we find that at room temperature  $\lambda_D/l_s \geq 1$ , and band transport breaks down for  $\mu \leq 90$  cm<sup>2</sup>/V s. The measured values of the hole mobility in barium titanate range from  $10^{-4}$  to  $0.5$  cm<sup>2</sup>/V s (Ref. 3), so a band-transport model is clearly not valid.

A more appropriate model considers the carriers to be localized “small” polarons. A polaron is formed when a free electron or hole is injected into a polarizable lattice. The lattice distorts around the free carrier and traps it, forming a polaron. The steady-state mobility of the small polaron is given for a one-dimensional case by<sup>15</sup>

$$\mu = (ea^2\omega/2\pi k_B T)P \exp(-E_A/k_B T), \quad (4)$$

where  $\omega$  is the vibration frequency of the lattice,  $e$  is electric charge, and  $a$  is the lattice constant. The activation energy  $E_A$  is the minimum energy required to strain the neighboring

lattice sites and dig a potential well of appropriate depth for the hole. The jump probability (given the coincidence of energy levels) is given by<sup>15</sup>

$$P = 2\pi J^2/\hbar\omega(4E_A k_B T/\pi)^{1/2}, \quad (5)$$

where  $J$  is the overlap integral of the electronic wave functions of the adjacent sites. Girshberg, Bursian, and Grushevsky<sup>13</sup> explained the anisotropy of electron mobility based on the anisotropy of the overlap integral in Eq. (5). They reasoned that for highly localized wave functions the overlap integral can be assumed to vary exponentially with the lattice constant:

$$J \propto e^{-a/r_0}, \quad (6)$$

where  $r_0$  is the localization length. Using this form in Eqs. (4) and (5) for the steady-state mobility of the small polaron, the ratio of mobilities is given by

$$\frac{\mu_{\parallel}}{\mu_{\perp}} \propto \frac{J_{\parallel}^2}{J_{\perp}^2} \propto \exp\{2[a_{\perp}(T) - a_{\parallel}(T)]/r_0\}. \quad (7)$$

At room temperature the lattice constant of tetragonal barium titanate parallel to the  $c$  axis is larger than that perpendicular to the  $c$  axis, so by this argument one expects the mobility parallel to the  $c$  axis to be smaller than the mobility perpendicular to the  $c$  axis, in agreement with measurements by Berglund and Baer<sup>12</sup> and Girshberg, Bursian, and Grushevsky<sup>13</sup> as well as our direct measurements of photoconductivity. As the temperature increases toward the tetragonal-to-cubic phase transition, the barium titanate lattice shrinks parallel to the  $c$  axis and stretches perpendicular to the  $c$  axis (and approaches a cube at the transition temperature  $T \sim 125$  °C). According to Eq. (6) the wave-function overlap and the resulting mobility should increase parallel to the  $c$  axis and decrease perpendicular to the  $c$  axis. This is also in agreement with our data. Despite its appeal and apparent agreement with experimental results, this argument leads to an unphysical localization length. The lattice constants of barium titanate parallel and perpendicular to the  $c$  axis differ by  $\sim 0.04$  Å at room temperature. For this difference to produce the measured hole mobility anisotropy of 20, we calculate from Eq. (7) that the polaron would have to be localized to  $r_0 \sim 0.03$  Å.<sup>12</sup> This seems unphysical, considering that the ionic radii are at least an order of magnitude larger in barium titanate. In addition this reasoning does not relate mobility to the dielectric constant. It is curious, however, that the dependence in Eq. (7) closely matches our measurements of mobility anisotropy as a function of temperature.

Another plausible, yet unsatisfactory, explanation uses the dependence of the overlap integral in Eq. (5) on the nuclear motion of adjacent sites.<sup>16,17</sup> At any finite temperature the lattice is in motion about its equilibrium position, making excursions of order

$$\Delta x \sim \sqrt{2k_B T/M\omega_0^2}, \quad (8)$$

where  $M$  is the reduced mass of the vibrating ions. The larger the excursions, the closer adjacent lattice sites are on the average, and the larger become the overlap integral and the mobility. The connection to the dc dielectric constant

becomes evident, for the larger dielectric constant,  $\epsilon_{\perp}$ , at room temperature corresponds to a smaller TO-phonon frequency, larger excursion, and higher mobility. Using  $\hbar\omega_{0\perp} \sim 4$  meV at room temperature for the TO phonon polarized perpendicular to the  $c$  axis,<sup>18</sup> and taking an average ion mass  $M \sim 3 \times 10^{-25}$  kg in Eq. (8) gives an estimate for the rms ionic displacement of  $\Delta x \sim 0.27$  Å. This value may be an overestimate. (It is difficult to imagine the ionic displacement exceeding the 0.04-Å anisotropy of barium titanate at room temperature.) Nevertheless, to explain the mobil-

ity anisotropy, the small polaron would have to be localized to 0.27 Å, which is still too small. Typical small polaron radii are on the order of the lattice constant,<sup>17</sup> here  $\sim 4$  Å. The resolution of this mystery awaits further theoretical and experimental investigation.

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considerations show that the dielectric constant that appears in the photorefractive grating equations,  $\epsilon^{\text{PR}}$ , is not necessarily the same as the unclamped value,  $\epsilon^{\text{UC}}$ , obtained from an external, uniform dc field measurement. [M. Zgonik, K. Nakagawa, and P. Günter, *J. Opt. Soc. Am. B* **12**, 1416 (1995).] For photorefractive grating wave vectors oriented perpendicular to the  $c$  axis, the two are expected to be the same;  $\epsilon_{\perp}^{\text{PR}} = \epsilon_{\perp}^{\text{UC}}$ . However, for gratings oriented parallel to the  $c$  axis the photorefractive dielectric constant is predicted to be nearly midway between the clamped and unclamped values;  $\epsilon_{\parallel}^{\text{PR}} \sim 0.63\epsilon_{\parallel}^{\text{UC}}$  at room temperature. This suggests that our electrically measured values of  $R$  (shaded region of Fig. 4) should be 63% lower than the holographic ones (at  $T = 20$  °C). But our data do not support this; our holographic and electrical values of  $R$  are the same to within the uncertainty of our measurements.

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